Institute of Science and Technology Austria

Boundary Superconductivity in the BCS Model

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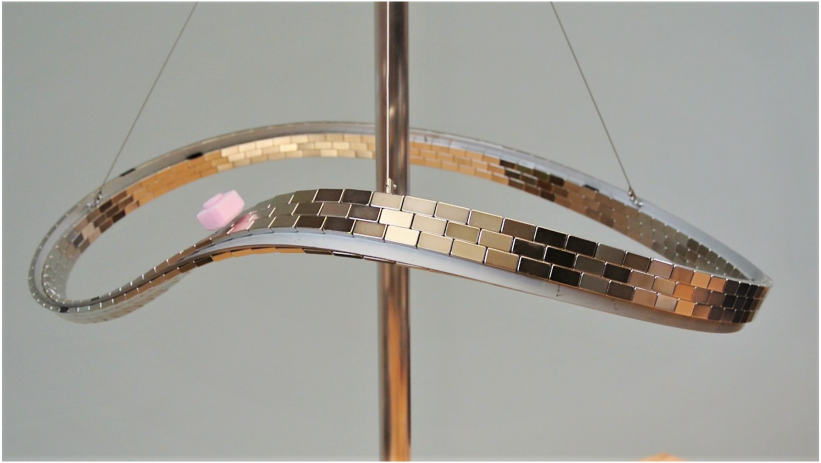
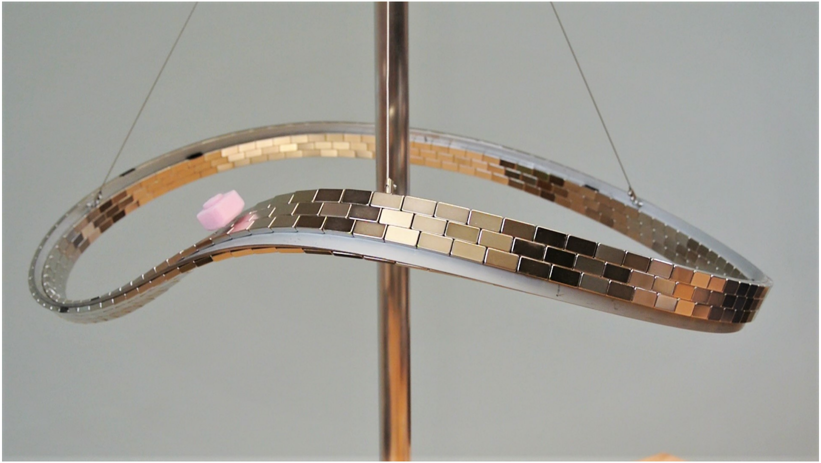
Based on joint work with Christian Hainzl and Robert Seiringer

arXiv:2201.08090 (to appear in Journal of Spectral Theory)



Source: https://www.experimente.physik.uni-freiburg.de/E_Elektrizitaet_und_Magnetismus/dasmagnetischefeld/kraefteimmagnetfeld/supraleiteraufmoebiusband



BCS theory

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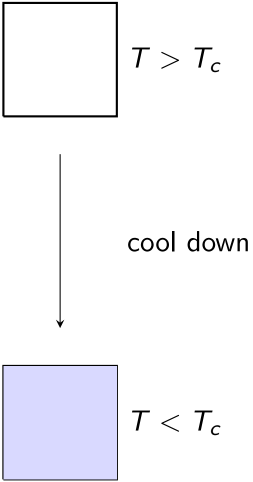
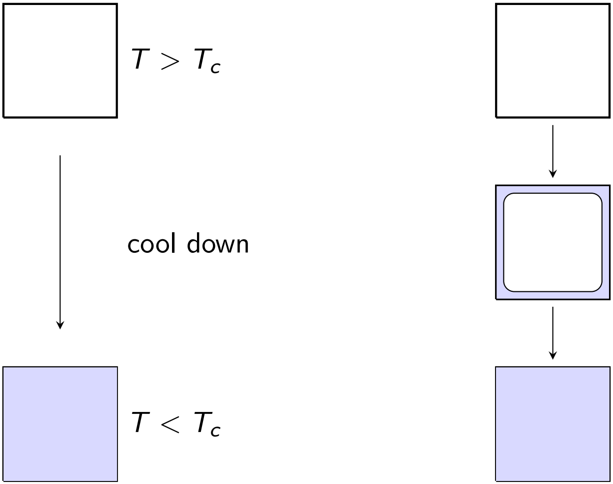


[Abrikosov 1964, De Gennes 1964]



[Abrikosov 1964, De Gennes 1964] [Samoilenka - Babaev 2020]

BCS model

BCS functional $\mathcal{F}_{T,\mu}(\gamma, \alpha)$:

Superconductivity $\Leftrightarrow$ $\alpha \not\equiv 0$ for minimizer (γ, α)

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Is this solution the minimizer?

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Is this solution the minimizer?

compute Hessian:

Two-body operator

$$H_T^\Omega = \frac{-\Delta_x - \Delta_y - 2\mu}{\tanh\left(\frac{-\Delta_x - \mu}{2T}\right) + \tanh\left(\frac{-\Delta_y - \mu}{2T}\right)} + \lambda V(x-y)$$

acting in $L^2_{symm}(\Omega \times \Omega)$.

Properties of H_T^Ω

$\inf \sigma(H_T^\Omega) < 0 \Rightarrow$ solution with $\alpha = 0$ not minimizer $\Rightarrow$ superconductivity

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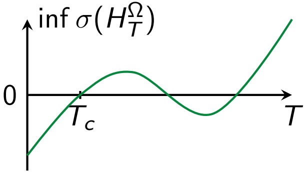
Critical temperature:



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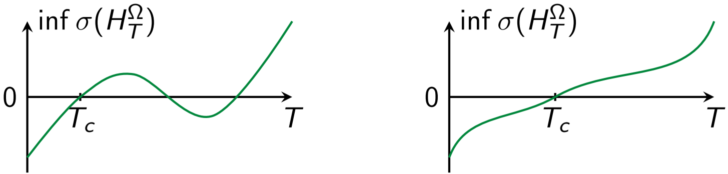

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translation invariant system: $T < T_c^\Omega \Leftrightarrow \inf \sigma(H_T^\Omega) < 0 \Leftrightarrow$ superconductivity [Hainzl-Hamza-Seiringer-Solovej, 2008]

Result

Compare $\Omega = \mathbb{R}_+$ with $\Omega = \mathbb{R}$ for $V = -\delta$.

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Theorem [Hainzl-R.-Seiringer 2022]

Let $\mu > 0$ and assume Dirichlet boundary conditions on $\mathbb{R}_+$.

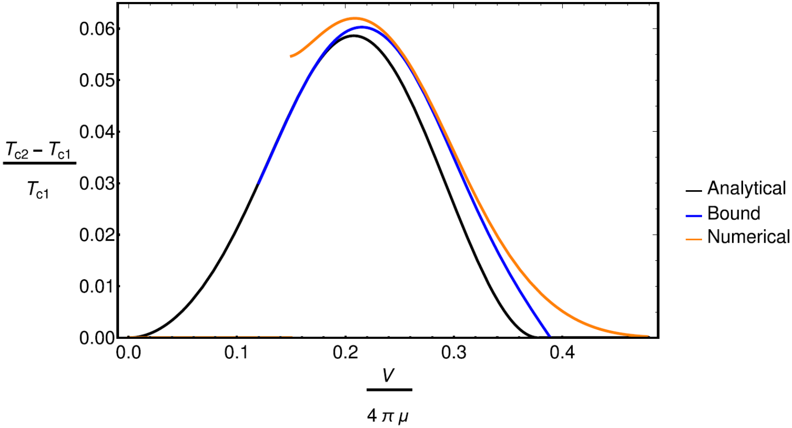
- There is a $\tilde{\lambda} > 0$ such that for $0 < \lambda < \tilde{\lambda}$

$$T_c^{\mathbb{R}_+}(\lambda) > T_c^{\mathbb{R}}(\lambda).$$

- In the weak and strong coupling limits

$$\lim_{\lambda \to 0,\infty} \frac{T_c^{\mathbb{R}_+}(\lambda) - T_c^{\mathbb{R}}(\lambda)}{T_c^{\mathbb{R}}(\lambda)} = 0.$$

Dependence on coupling [Samoilenka - Babaev 2020]



Neumann boundary

Theorem [Hainzl-R.-Seiringer 2022]

Let $\mu > 0$ and assume Neumann boundary conditions on $\mathbb{R}_+$.

- For all $\lambda > 0$
$$T_c^{\mathbb{R}_+}(\lambda) > T_c^{\mathbb{R}}(\lambda).$$
- In the weak coupling limit
$$\lim_{\lambda \to 0} \frac{T_c^{\mathbb{R}_+}(\lambda) - T_c^{\mathbb{R}}(\lambda)}{T_c^{\mathbb{R}}(\lambda)} = 0.$$
- In the strong coupling limit
$$0 < \lim_{\lambda \to \infty} \frac{T_c^{\mathbb{R}_+}(\lambda) - T_c^{\mathbb{R}}(\lambda)}{T_c^{\mathbb{R}}(\lambda)} < \infty.$$

References

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- P. G. De Gennes. Boundary Effects in Superconductors. *Reviews of Modern Physics*, 36(1):225–237, Jan. 1964.
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- C. Hainzl, B. Roos and R. Seiringer. Boundary Superconductivity in the BCS Model. *arXiv:2201.08090 [math-ph, cond-mat.supr-con]*, Jan. 2022.
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BCS model

BCS functional

$$\mathcal{F}(\Gamma) = \mathrm{Tr}\,(h-\mu)\gamma + T\,\mathrm{Tr}\,\Gamma\ln\Gamma + \int\int_{\Omega\times\Omega} |\alpha(x,y)|^2 V(x-y)\mathrm{d}x\mathrm{d}y$$

- $\Gamma = \begin{pmatrix} \gamma & \alpha \\ \overline{\alpha} & 1-\overline{\gamma} \end{pmatrix}$ self-adjoint operator on $L^2(\Omega) \oplus L^2(\Omega)$
- 1-particle density matrix: $\langle\phi|\gamma\psi\rangle = \mathrm{Tr}\; a^\dagger(\psi)a(\phi)\rho$
- pairing expectation: $\langle\phi|\alpha\psi\rangle = \mathrm{Tr}\; a(\psi)a(\phi)\rho$

Minimize $\mathcal{F}(\Gamma)$ over $0 \leq \Gamma \leq 1$.

Superconductivity $\Leftrightarrow$ $\alpha \not\equiv 0$ for minimizer

Proof strategy

$V = -\delta$, $\Omega = \mathbb{R}, \mathbb{R}_+$ Dirichlet b.c. Want: $T_c^{\mathbb{R}_+}(\lambda) > T_c^{\mathbb{R}}(\lambda)$

Birman-Schwinger principle:

$$H_T^\Omega = K_T - \lambda\delta \quad \leftrightarrow \quad A_T^\Omega = \delta^{1/2} K_T^{-1} \delta^{1/2}$$

- get rid of boundary conditions
- reduce number of variables
- $\inf \sigma(H_T^\Omega) < 0 \Leftrightarrow \sup \sigma(A_T^\Omega) > \frac{1}{\lambda}$

Construct trial state such that $\dfrac{\langle \psi, A_T^{\mathbb{R}_+} \psi \rangle}{\langle \psi, \psi \rangle} > \sup \sigma(A_T^{\mathbb{R}})$

$\rightarrow$ works in weak coupling limit

Momentum space representation of A_T

For $\psi \in L^2(\mathbb{R})$

$$A_T^{\mathbb{R}}\psi(p) = \int_{\mathbb{R}} B_{T,\mu}(p,q)\psi(p)\mathrm{d}q$$

and

$$A_T^{\mathbb{R}_+}\psi(p) = \int_{\mathbb{R}} B_{T,\mu}(p,q)(\psi(p) - \psi(q))\mathrm{d}q$$

where

$$B_{T,\mu}(p,q) = \frac{1}{4\pi}\frac{\tanh\left(\frac{\left(\frac{p+q}{2}\right)^2-\mu}{2T}\right) + \tanh\left(\frac{\left(\frac{p-q}{2}\right)^2-\mu}{2T}\right)}{\left(\frac{p+q}{2}\right)^2 + \left(\frac{p-q}{2}\right)^2 - 2\mu}$$

Maximizer of $A_T^{\mathbb{R}_+}$ [Samoilenka - Babaev 2020]